Cloud-Assisted Security Services (CloSer)
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Private Graph Search

- **Introduction**: A Private Graph Search protocol enables users to query the graph to find if there is a path from node A to node B without revealing their queries. Users are also prevented from learning anything else about the graph.

- **Protocol**: The database owner holds two lists of triplets: (sourceuser, sourcehost, fingerprint) and (fingerprint, sourceuser, sourcehost). These define trust relations between users on different hosts. This database can be illustrated as a directed graph.

- **Queries on Transitive Closure**: We store the encrypted matrix in a cloud that is not allowed to know what bit is queried. There are three parties in this protocol: Owner of the graph, the User and the Cloud.

**Queries on Transitive Closure**

To encrypt the matrix:
- Choose hash $\mathcal{H}$ and $N = pq$ and some $y \in QR_N$ such that $\text{Jacobi}(y,N) = 1$
- For every index $i$, find the smallest $j$ such that $\text{Jacobi}(\mathcal{H}(j||i),N) = 1$
- Encrypt matrix, in such a way that if $\mathcal{H}(j||i) \in QR_N$, flip the bit, otherwise keep it.
- Send $\mathcal{H}$, $N$ and $y$ to the user

To decrypt bit $i$:
- Find smallest $j$ such that $z = \mathcal{H}(j||i)y^2y^q \mod N$
- Ask owner if $z$ or $zy \in QR_N$
- Flip bit $i$ if needed

**Conclusion**: We use Goldwasser-Micali and Paillier homomorphic encryption schemes in our protocol. After executing the protocol, user obtains the value of just one bit in the matrix. Cloud does not learn the graph nor the user’s query, and the graph owner does not learn the nodes that the user is interested in.